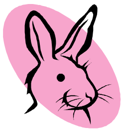
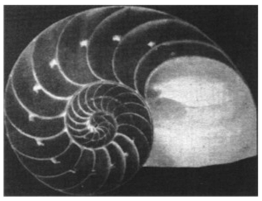
Rabbits and Pinecones





1, 2, 3, 5, 8, 13, 21, 34, 55, 89 …

1

2

- **Boethius**
  - ***"Everything which since the beginning of things was produced by Nature seems to be formed according to numerical relations, issued from the wisdom of the Creator".***
- **Pythagoras**
  - ***"Everything is arranged according to number."***
- **Plato**
  - ***"Numbers are the highest degree of knowledge."***
  - ***"Number is knowledge itself."***
  - ***"And it was then that all these kinds of things thus established received their shapes from the Ordering One, through the action of Ideas and Numbers."***

3

- **Sir Isaac Newton:**
  - ***"It is not to be conceived that mere mechanical causes could give birth to so many regular motions … this most beautiful system of the Sun, planets and comets, could only proceed from the counsel and domination of an intelligent and powerful Being".***
- **Dr Roles:**
  - ***"see the existence of a plan and experience the presence of the designer first hand".***

4

Fibonacci

- 1175-1250 (approx.)
- Leonardo of Pisa
- Son of Bonacci = Fi'Bonacci
- Also known as Bigolo = 'well-travelled'
- Introduced the Arabic/Hindu numerals and the zero

5

Arithmetic with Roman Numerals

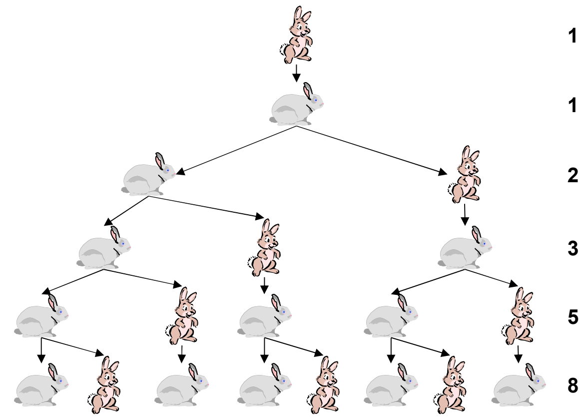
- II + I = III
- X + II = XII
- CLXXIIII + XXVIII = CCII !
- XIX times XXIV = ?
- The introduction of Arabic/Hindu numerals and the symbol for zero simplified these operations

6

Number Series

	1	2	3	4	5	6	7	8	
Squares	**1**	**4**	**9**	**16**	**25**	**36**	**49**	**64**	…
Powers of 2	**1**	**2**	**4**	**8**	**16**	**32**	**64**	**128**	…

7



8

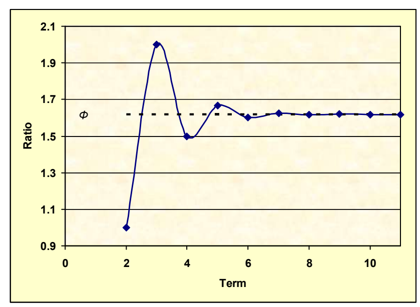
Fibonacci Series

- Each term is the sum of the preceding two
- Every 3rd term divisible by 2 (the 3rd term)
- Every 4th term divisible by 3 (the 4th term) etc.
- Ratios of consecutive terms converge to the golden proportion

Term	Series	Ratio	Inverse Ratio
1	1		
2	1	1	1
3	2	2	0.5
4	3	1.5	0.666666667
5	5	1.666666667	0.6
6	8	1.6	0.625
7	13	1.625	0.615384615
8	21	1.615384615	0.619047619
9	34	1.619047619	0.617647059
10	55	1.617647059	0.618181818
11	89	1.618181818	0.617977528
12	144	1.617977528	0.618055556
13	233	1.618055556	0.618025751
14	377	1.618025751	0.618037135
15	610	1.618037135	0.618032787
16	987	1.618032787	0.618034448
17	1597	1.618034448	0.618033813
18	2584	1.618033813	0.618034056
19	4181	1.618034056	0.618033963
20	6765	1.618033963	0.618033999

9

Series	Ratio
1	
1	1
2	2
3	1.5
5	1.666666667
8	1.6
13	1.625
21	1.615384615
34	1.619047619
55	1.617647059
89	1.618181818



1.61803398875…

10

Algebraic Properties of ϕ

1	1
ϕ	ϕ
$\phi^2 = \phi + 1$	$\phi^2 = \phi + 1$
$\phi^3 = \phi^2 + \phi$	$\phi^3 = 2\phi + 1$
$\phi^4 = \phi^3 + \phi^2$	$\phi^4 = 3\phi + 2$
$\phi^5 = \phi^4 + \phi^3$	$\phi^5 = 5\phi + 3$
$\phi^6 = \phi^5 + \phi^4$	$\phi^6 = 8\phi + 5$
$\phi^7 = \phi^6 + \phi^5$	$\phi^7 = 13\phi + 8$
…	…

11

Golden Ratio

$$\phi = \frac{\sqrt{5}+1}{2} = 1.61803398875\ldots$$

$$\frac{1}{\phi} = \phi - 1 = \frac{\sqrt{5}-1}{2} = 0.61803398875\ldots$$

Solutions of $x^2 = x + 1$ are ϕ and $^1/_\phi$

$$\phi = \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\ldots}}}}}$$

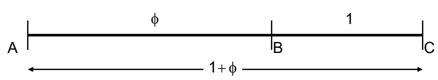
12

13

- **Luca Pacioli:**
  The Divine Proportion
- **Johannes Kepler:**
  Geometry has two great treasures: one is the theorem of Pythagoras; the other, the division of a line into extreme and mean ratio. The first we may compare to a measure of gold; the second we may name a precious jewel.

14

Division of a Line in the Golden Ratio



$$\frac{AB}{BC} = \frac{\phi}{1} = \frac{AC}{AB} = \frac{1+\phi}{\phi}$$

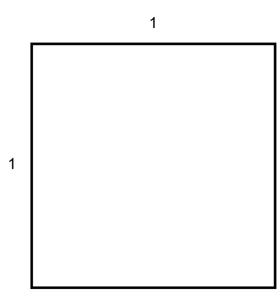
15

Geometric Construction of the Golden Rectangle

The following slides illustrate the steps for the geometric construction of the golden rectangle, as outlined in the presentation

17

Geometric Construction of the Golden Rectangle



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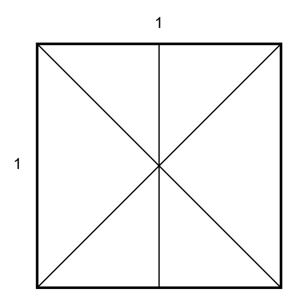
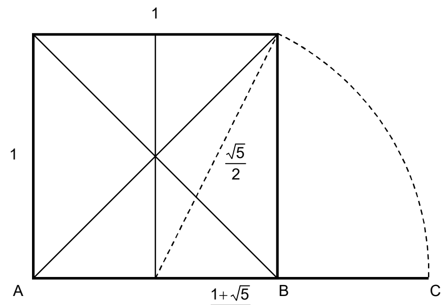
Geometric Construction of the Golden Rectangle



19

Geometric Construction of the Golden Rectangle

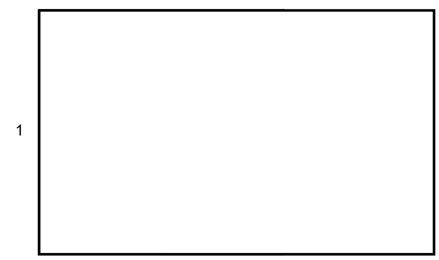


$\frac{\sqrt{5}}{2}$

$\frac{1+\sqrt{5}}{2}$

20

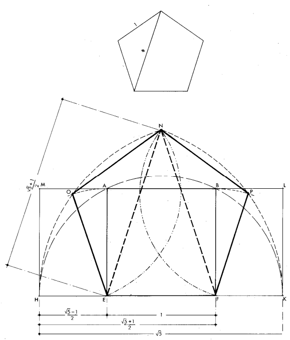
Geometric Construction of the Golden Rectangle



$\frac{1+\sqrt{5}}{2}$

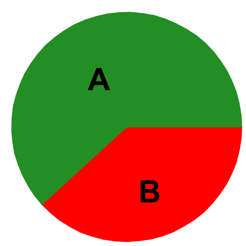
21

Pentagon Related to the Golden Ratio



22

Division of a Circle in the Golden Ratio



$$\frac{A}{\text{Whole}} = \frac{B}{A} = \phi - 1 = 0.61803...$$

$$\text{Angle A} = 222.49...\text{ deg.} = (\phi - 1) \times 360$$

23

Vedic Mathematics for Division of 1 by 19

One more than the one before = 2

$^{1}/_{19}$ = **0. 0 5 2 6 3 1 5 7 8**

(carries: 1 1 1 1 1 1)

9 4 7 3 6 8 4 2 1 …

(carries: 1 1 1)

24

Vedic Mathematics for Division of 11 by 89

One more than the one before = 9

$^{11}/_{89}$ = **0. 1 2 3 5 8 13 21 34 55 …**

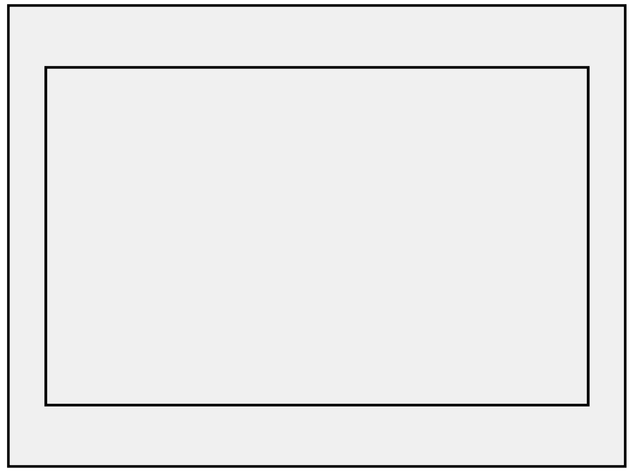
(2 3 5 8 13 21 34 55 89)

25

Division of the Golden Rectangle

The following slides illustrate the steps for dividing the golden rectangle and drawing a spiral, as outlined in the presentation

27



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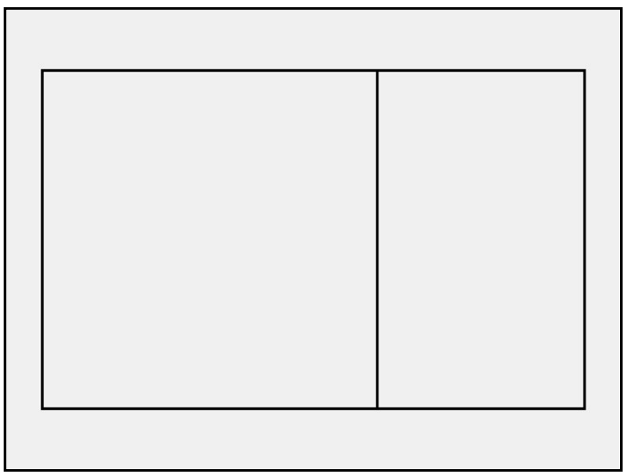

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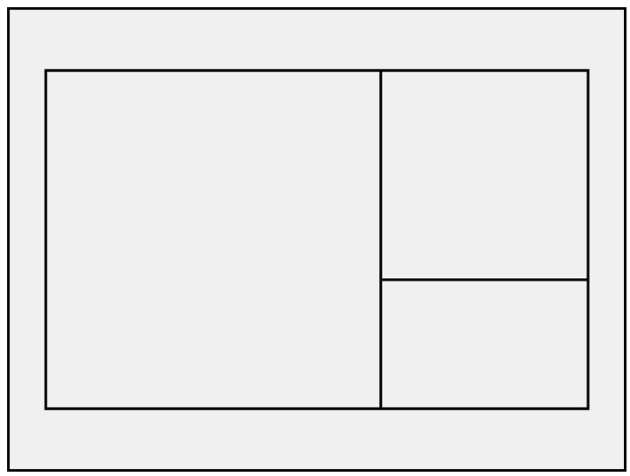

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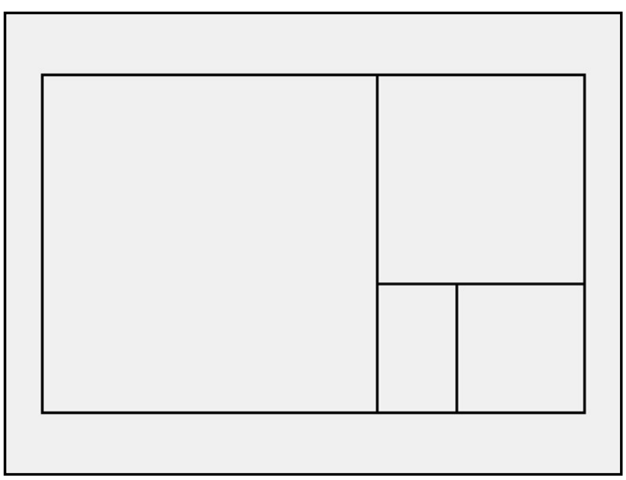

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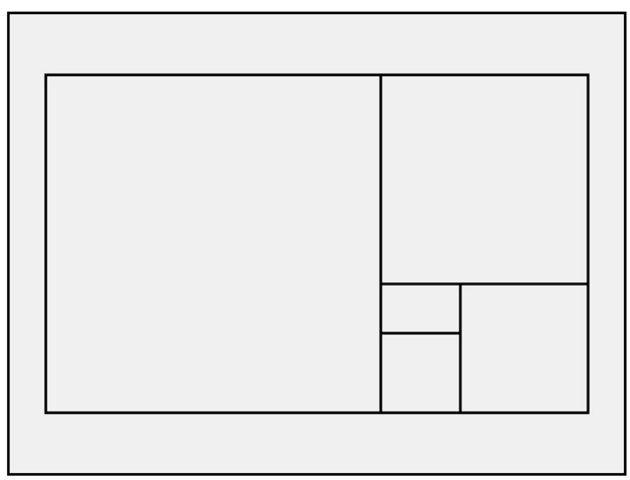

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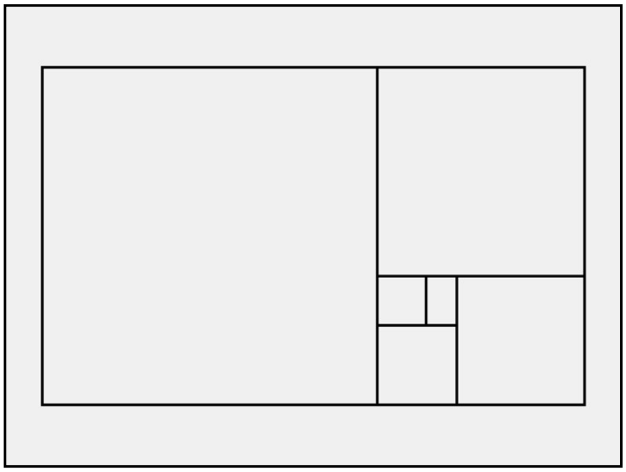

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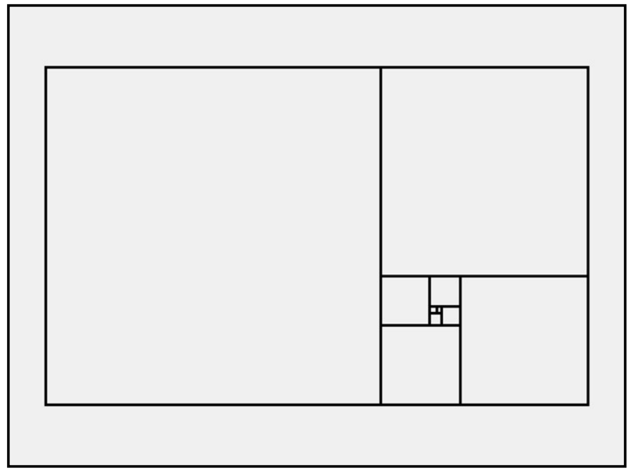

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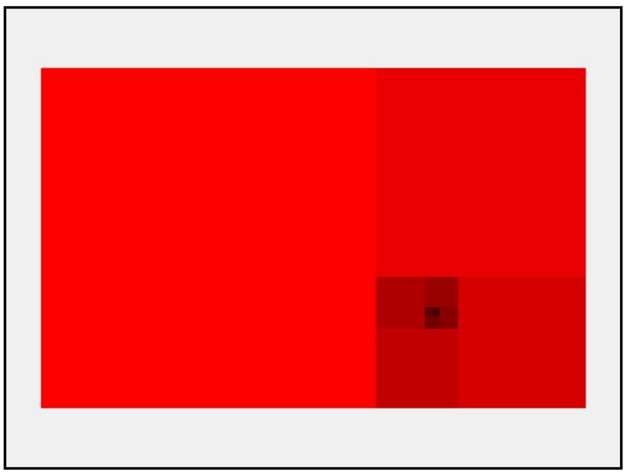

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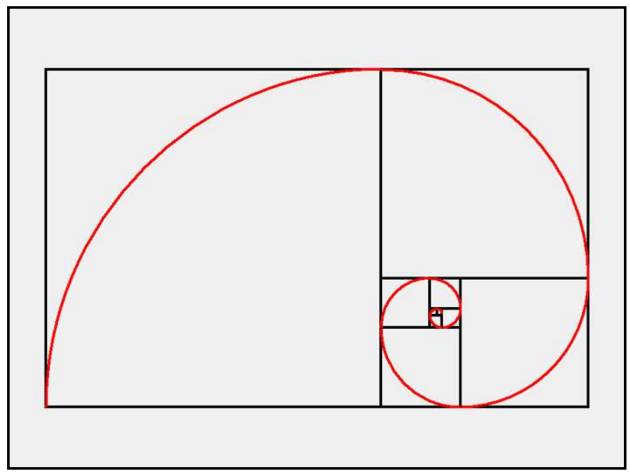

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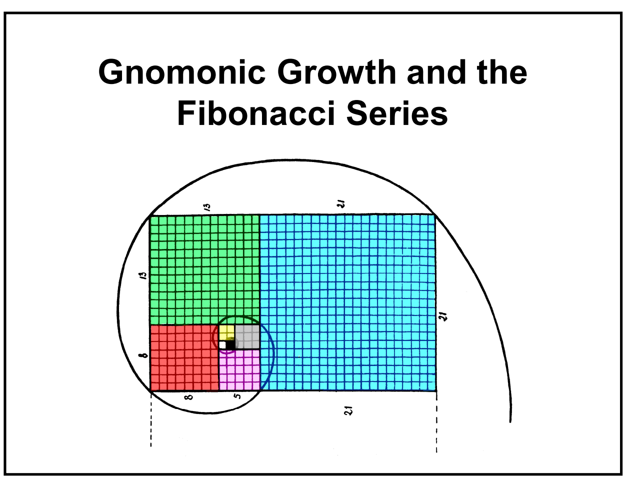

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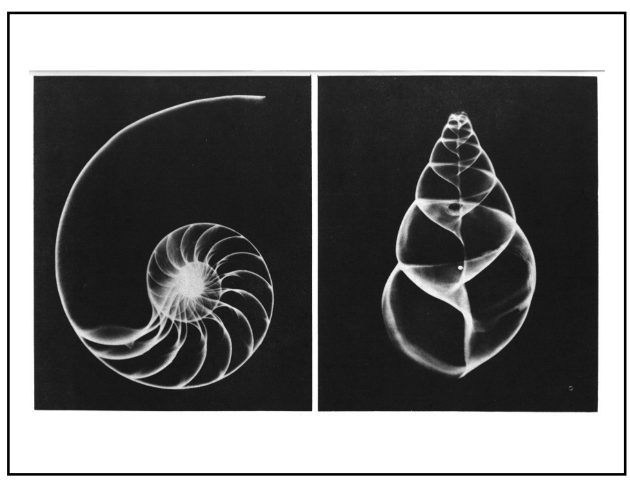


38

Gnomonic Growth

- Something added to a shape reproduces that shape on a larger scale
- Multiplication through addition
- The basis for growth of many animal forms

39

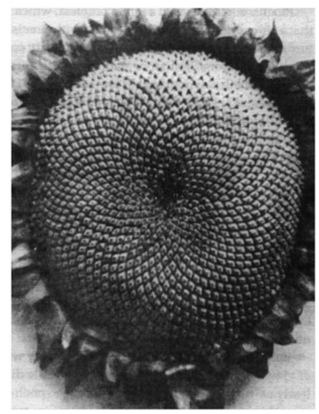
Fibonacci Numbers in Nature

- Petals in flowers
  - Lilies 3
  - Buttercups 5
  - Delphiniums 8
  - Marigolds 13
  - Asters 21
  - Daisies 34, 55, or 89

40

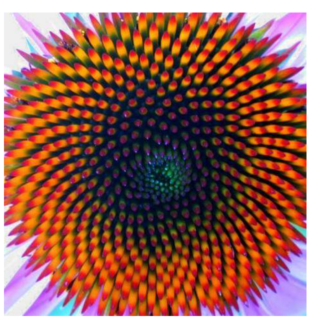
41

Typical spirals 89 one way and 144 the other



42

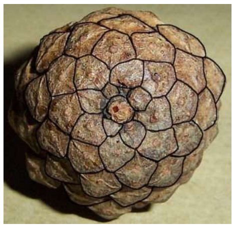
Coneflower

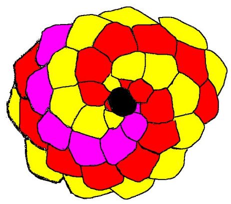


43

44

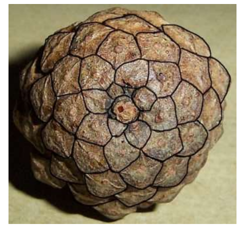
5 spirals clockwise

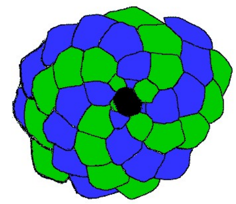




46

8 spirals anticlockwise





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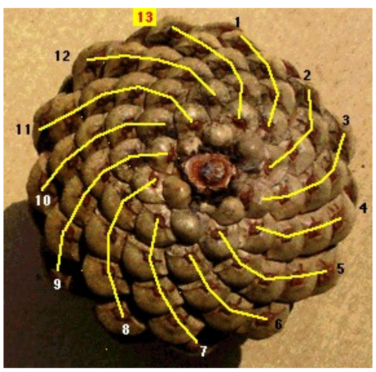

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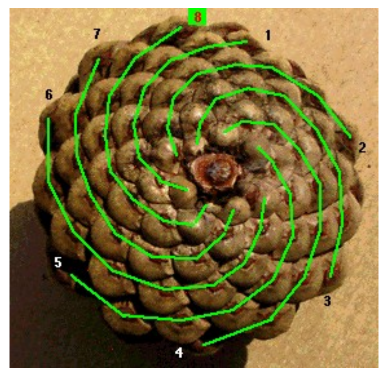
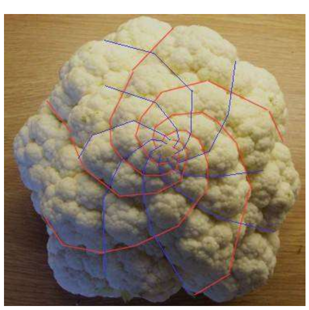


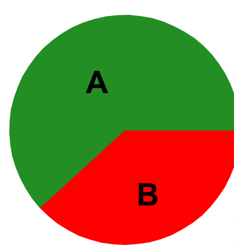
50

- Pinecones
- Pineapples
- Cacti
- Fruit
- Cauliflower



52

Division of a Circle in the Golden Ratio



$$\frac{A}{Whole} = \frac{B}{A} = \phi - 1 = 0.61803...$$

Angle A = 222.5 degrees (approx.)

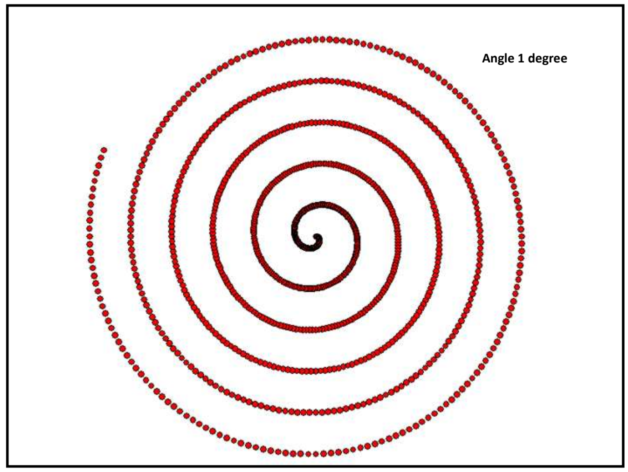
Angle B = 137.5 degrees (approx.)

53

Generation of Spirals and Seed Heads

The following slides illustrate the generation of various spirals and simulation of packing in seed heads, as outlined in the presentation

55



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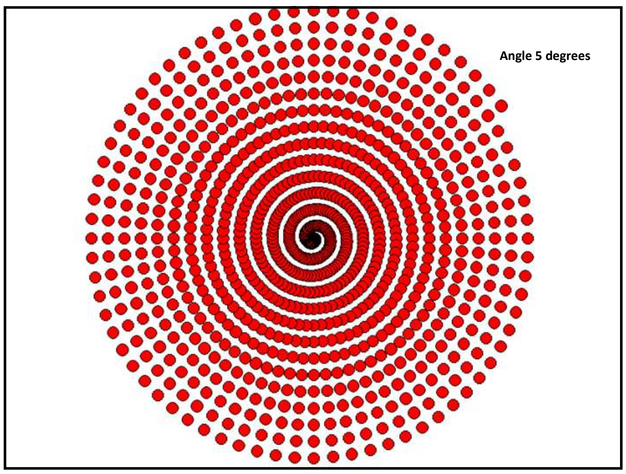

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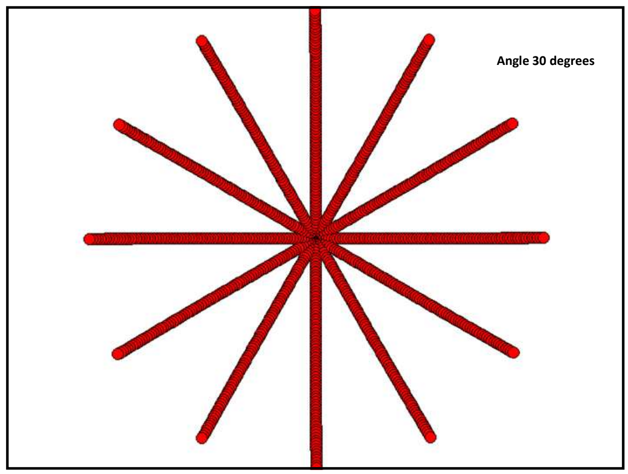

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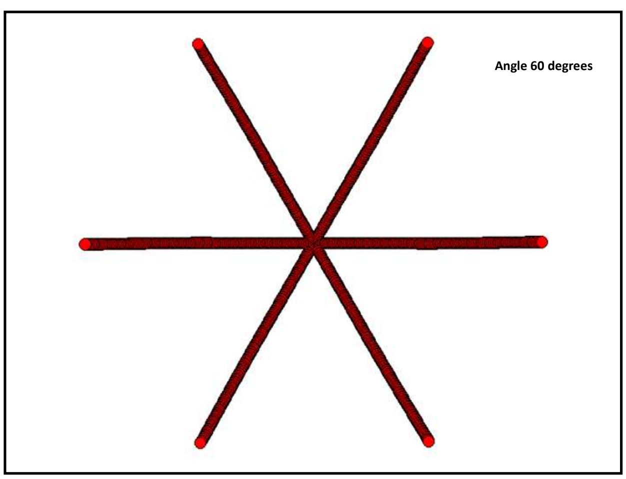

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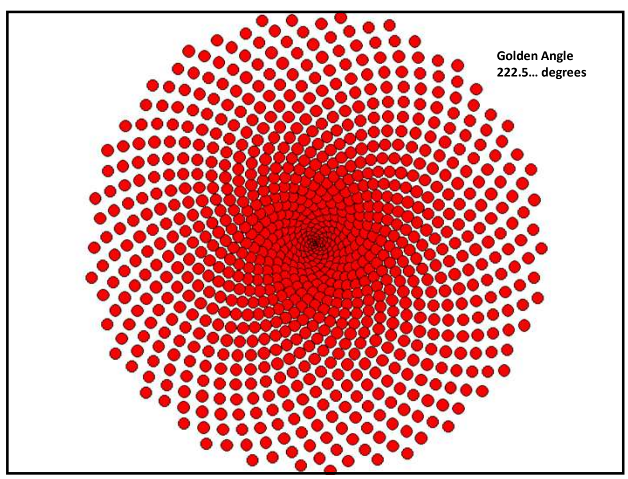

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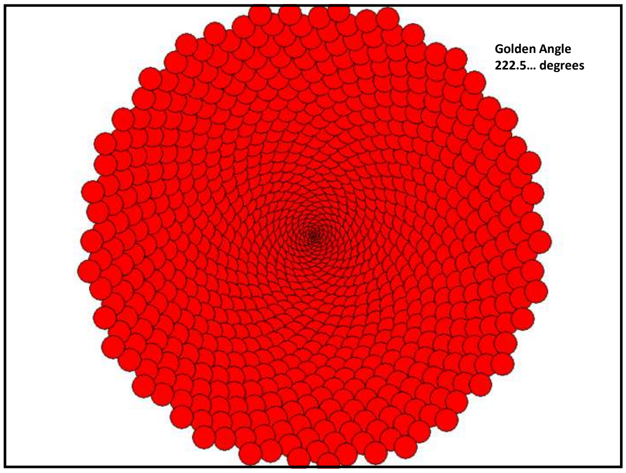

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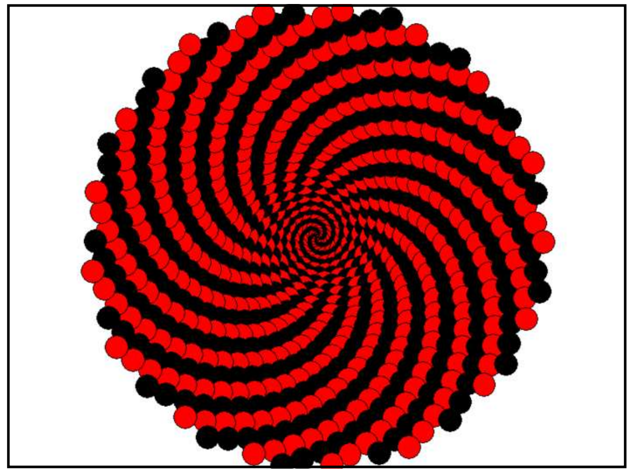

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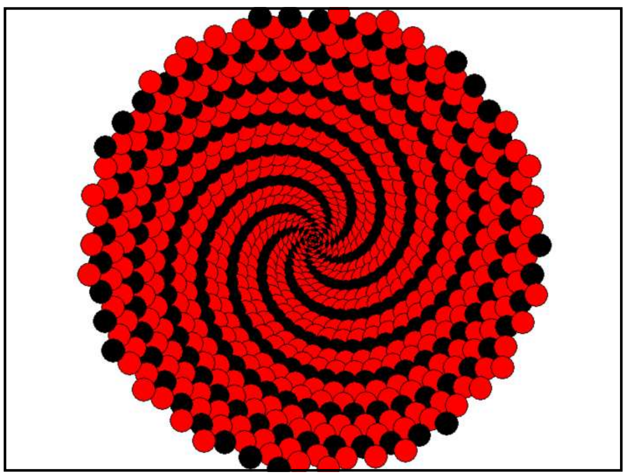

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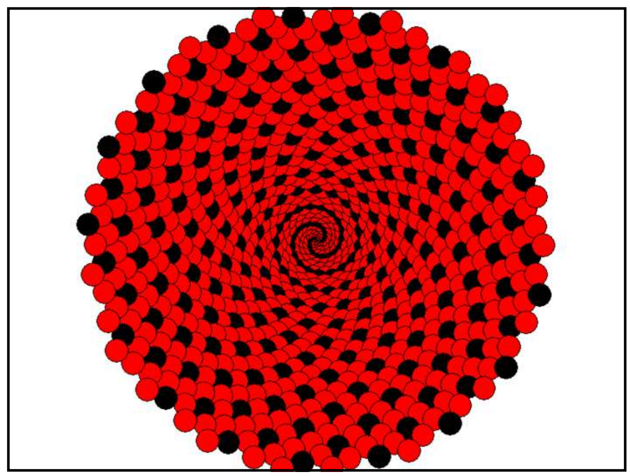

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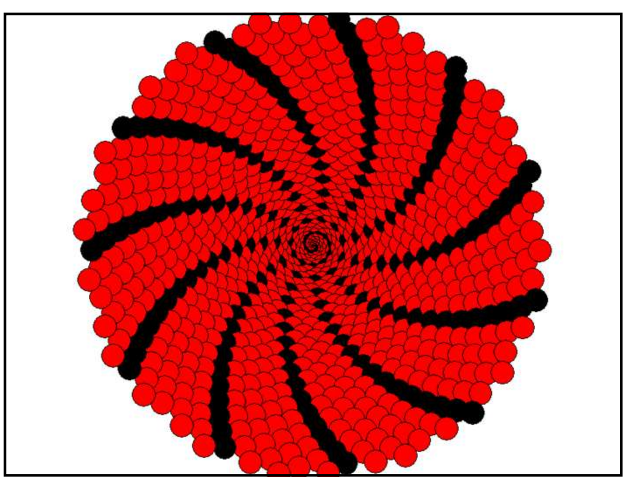

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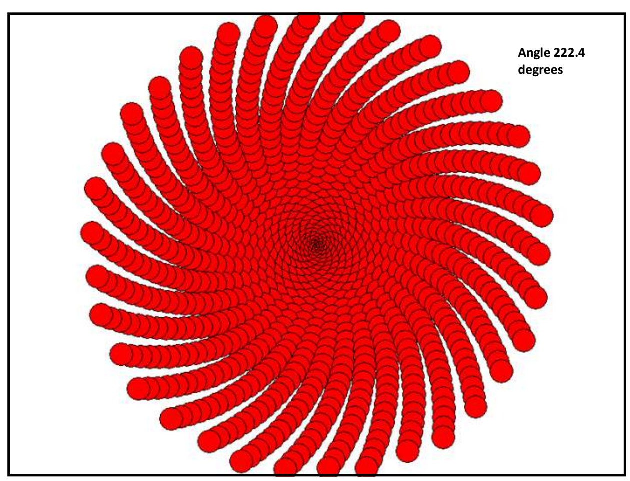

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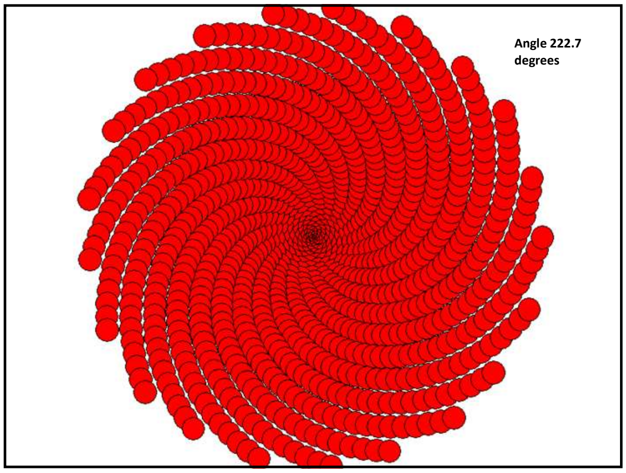

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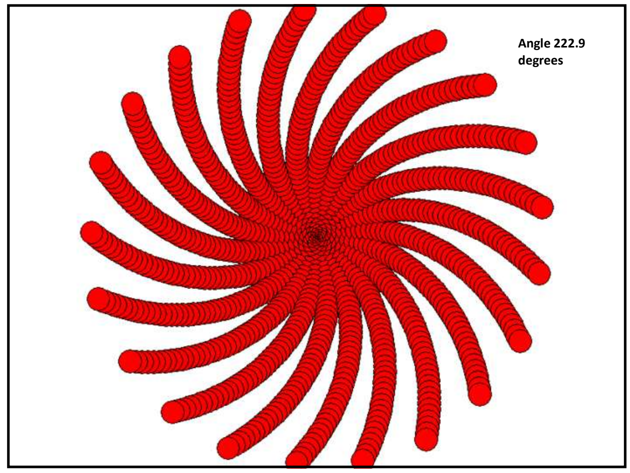

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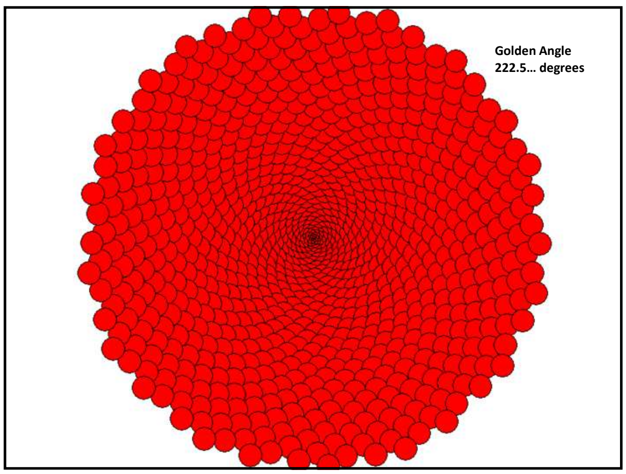

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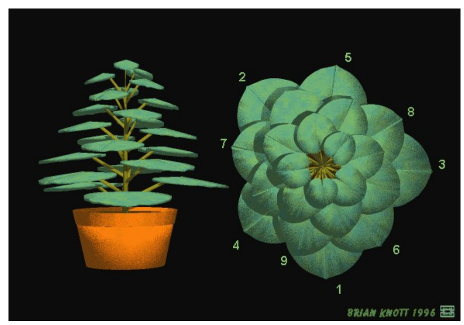
Computer Simulation of Plant Growth



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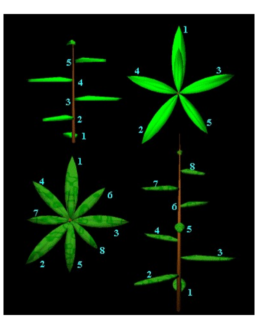
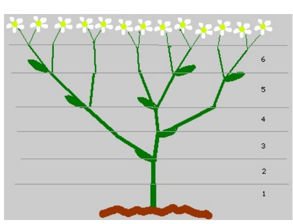
Computer Simulations of Plant Growth





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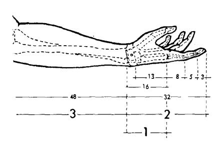
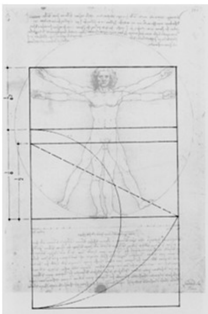
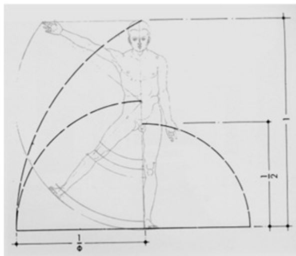


72

The Human Form Related to the Golden Ratio





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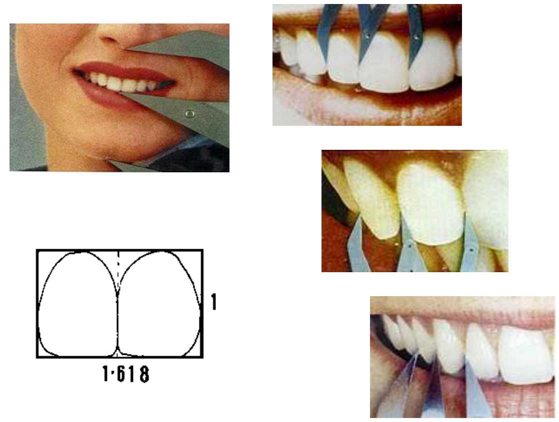

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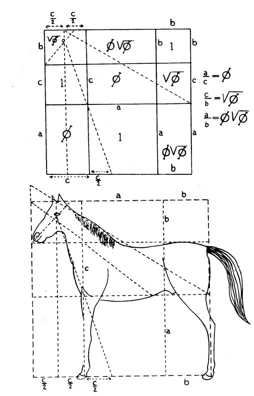

Plate XLII
Harmonic Analysis of Horse in Profile

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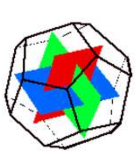
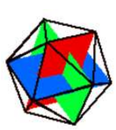
The Golden Rectangle and the Platonic Forms

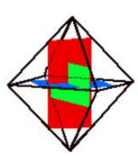
Dodecahedron



Icosahedron



Octahedron



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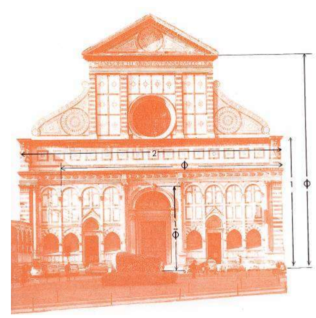
Building Design Based on the Golden Ratio



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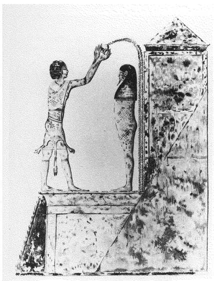
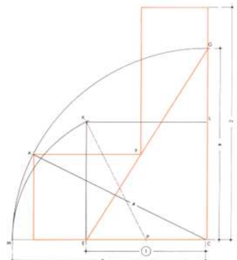
Egyptian Tomb of Petosiris





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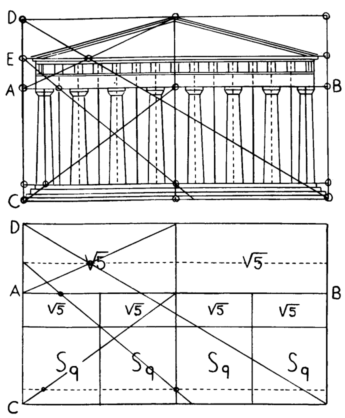

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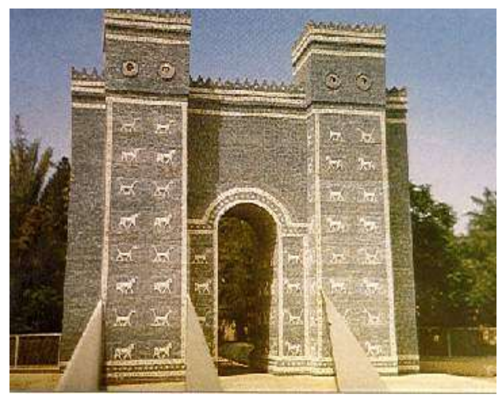

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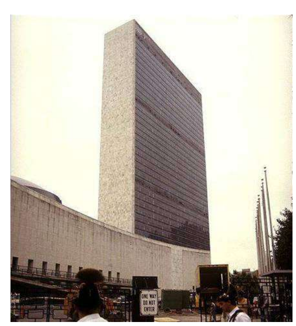
United Nations Building New York



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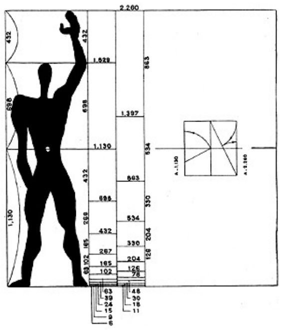
Le Corbusier – Modulor Scale



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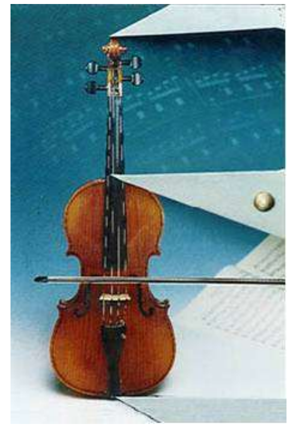

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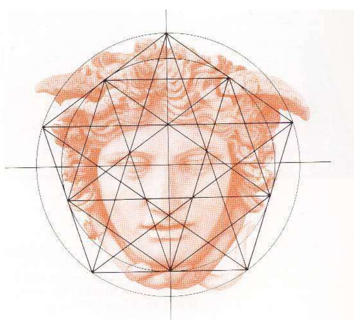
Mask of Hermes



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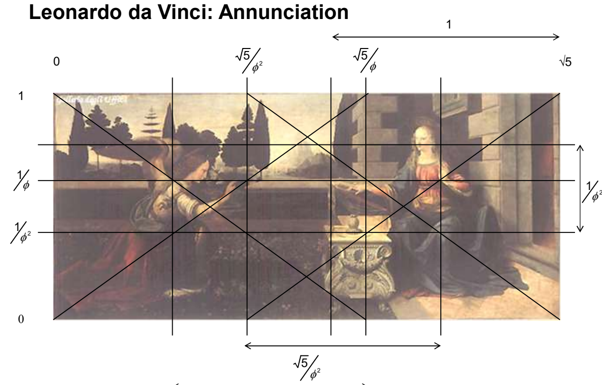
Leonardo da Vinci: Annunciation



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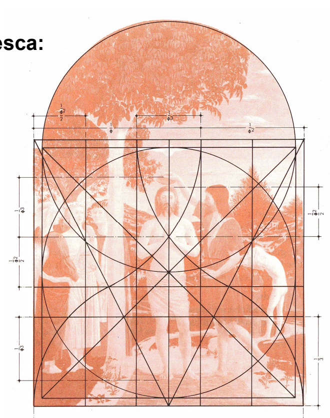
Piero Della Francesca:

Baptism of Christ



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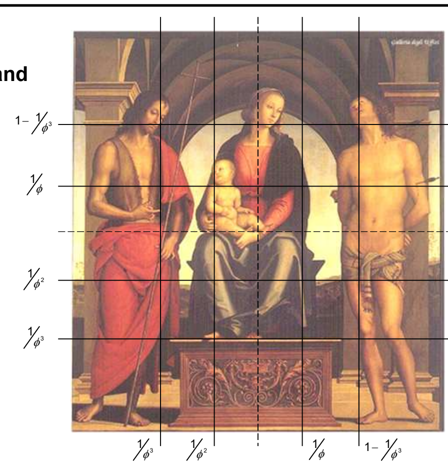
Perugino:

Madonna and Saints



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Albrecht Durer

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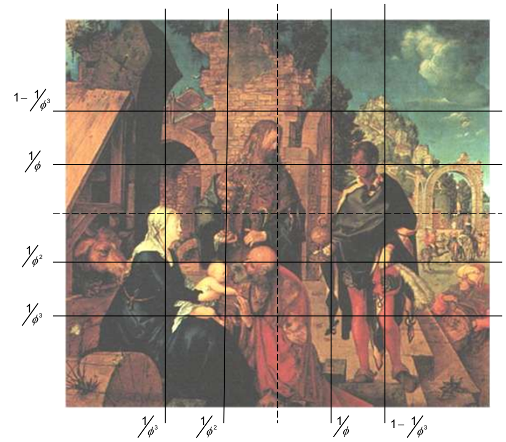

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Melancholia

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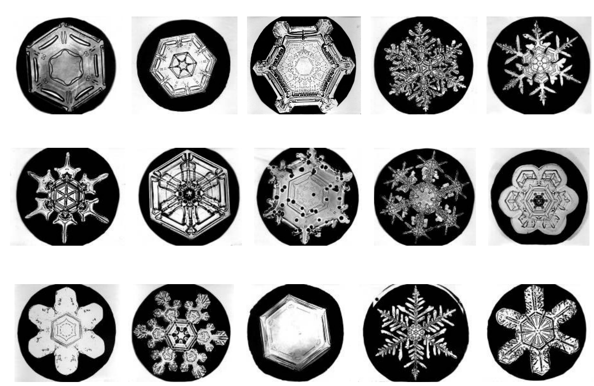

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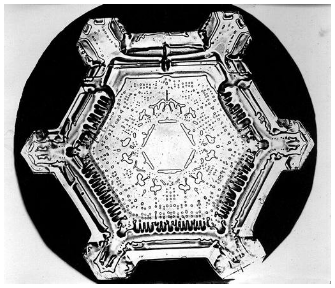
Wilson Bentley



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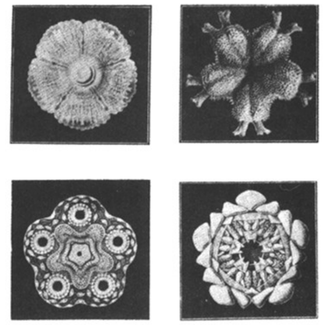
Five-fold Symmetry in Sea Creatures



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